A regime switching model for smart beta investing using hidden Markov models

Elizabeth Fons

University of Manchester, AllianceBernstein

2019 Quant Summit Europe





About this work

A novel dynamic asset allocation system using Feature Saliency Hidden Markow models for smart beta investing

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b. Development of the second secon These factor returns can be a scenar of risk and/or imported stress, nel understanding therhor say additional risk is nelrependent on the predicted state (Nyrmay et al., 200 rs). As an trenative approach is to dynamically optimize a portfolio using and so are factor anoming to superve perturbs second moves information from the information regime information to con-ther second approach and use the regime information to constrate to achieve these goals as endeated with by utilising a transpannet, cyclematic, rules based approach, beinging down allocation (DAA) system to conduct portfolios theraph a se-gime witching model and perform a systematic analysis using headered of conductivitient of factors to training the IPMM with the same factors that will be used for the attocation in the port folio. Our study shows that using the regime information for totas. Our study shows that using the regime adversation from the IDMM has a better areferences of the a single pasing other

A novel dynamic asset allocation system using Feature Saliency Hidden Markov models for smart beta investing https://arxiv.org/abs/1902.10849

Elizabeth Fons^{1,3} Paula Dawson¹ Jeffrey Yau^{1,2} Xiao-jun Zeng³ John Keane³

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Overview

Motivation

Part 1

- Hidden Markov Models
- Data
- DAA system and Implementation details
- Main results

Part 2

- Feature selection and HMMs
- Implementation details
- Main results
- Onclusions and future work

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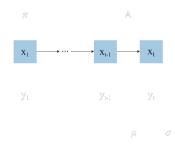
Two main contributions:

- Use HMMs to identify market regimes and to build the portfolios. We did a systematic study with multiple factors and types of portfolios.
- Output: Use an unsupervised feature selection algorithm to select optimal features for the HMM. We show that feature selection improves regime identification.

Unsupervised clustering method, very useful for sequential data, since it's able to handle temporal correlations.

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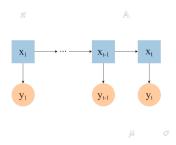
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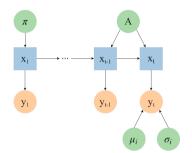
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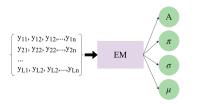
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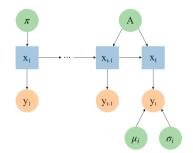
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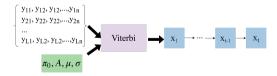
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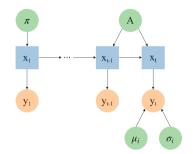
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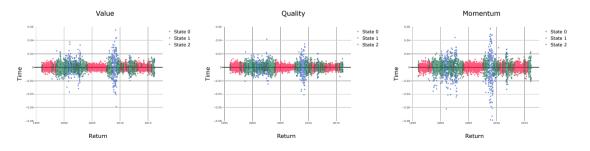


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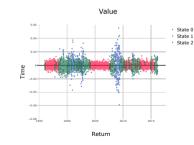


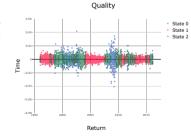
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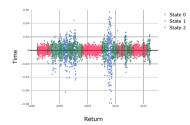
Regime switching models



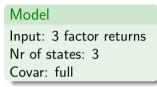


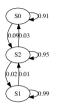




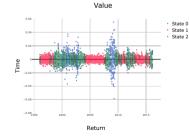


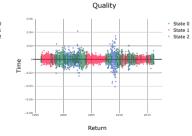
Momentum



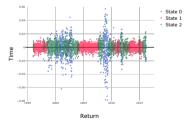


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Data

Daily factor data from S&P500

The universe is ranked, a portfolio is constructed with the top 20% of stocks and short positions in the bottom 20% of stocks.

MSCI USA enhanced indices

Start from 6 MSCI indices (Value, Momentum, Size, Volatility, High Yield, Quality) and subtract equally weighted index.

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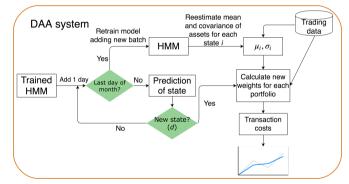
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Dataset	Date	Nr of features	Frequency		
Factor data	Jan-1988 to Feb-2016	25	Daily		
MSCI Enhanced	Jan-1999 to Feb-2016	6	Daily		

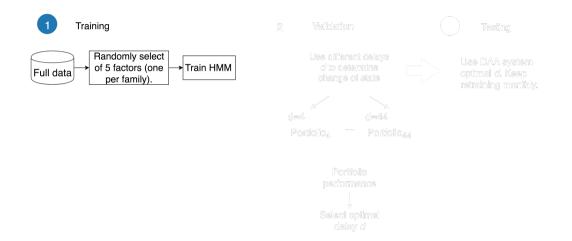
Dynamic Allocation System

Objective: build a DAA system incorporating information from a HMM and test it systematically.

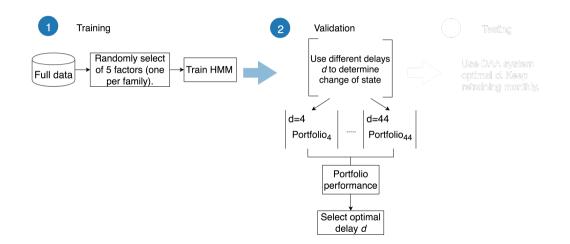
- State is determined daily, by adding one observation per day (yes, this is very noisy!).
- Given a change of state, use information from model (mean and variance) to recalculate the weights of the portfolio.
- After one month, add the observations to the stack of previous ones with an expanding window and retrain the model.



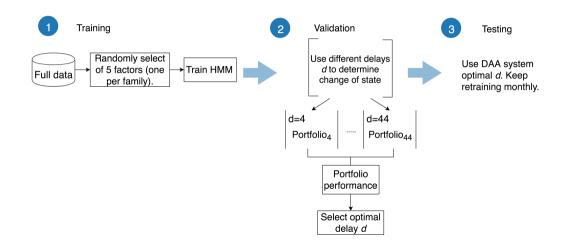
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- Test set is 4 years.
- The model is retrained monthly after the initial training, to account for changes in the parameters (to relax stationarity).

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Three groups of portfolios:

HMM Six portfolios built using information from HMMs.

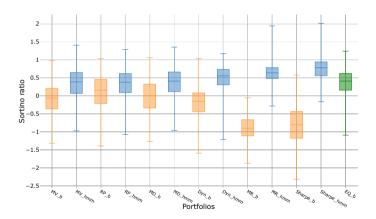
Benchmark 1 Six portfolios rebalanced monthly, single regime.

Benchmark 2 Equally weighted portfolio rebalanced monthly.

Total = (6 + 6 + 1) * 1260 portfolios.

Main results part 1

- Blue: HMM portfolios
- Orange: no-regime portfolios
- Green: EQ portfolio.



Main results part 1

	Ann ret	Ann vol	IR	Skw	kurt	D. risk	Sortino	DD	DD days
EQ	0.77	2.88	0.26	-0.14	0.81	2.05	0.37	379	318
Dyn HMM	1.67	4.73	0.34	-0.19	1.35	3.37	0.48	32	291
Dyn Bench	-0.60	3.98	-0.14	-0.40	1.68	2.96	-0.19	1136	682
Sharpe HMM	2.31	4.66	0.53	-0.19	1.16	3.29	0.75	429	253
Sharpe Bench	-3.14	4.89	-0.64	-0.79	4.49	3.80	-0.82	1375	873
MR HMM	3.19	7.03	0.46	-0.19	1.34	4.98	0.65	35	264
MR Bench	-5.03	7.20	-0.69	-0.78	3.71	5.63	-0.88	>4000	1001
MV HMM	0.61	2.41	0.24	-0.14	0.96	1.72	0.35	662	309
MV Bench	-0.12	2.24	-0.07	-0.11	0.83	1.61	-0.09	520	511
MD HMM	0.69	2.54	0.26	-0.14	1.01	1.80	0.37	340	306
MD Bench	0.01	2.39	-0.02	-0.12	0.84	1.71	-0.02	454	447
RP HMM	0.63	2.58	0.24	-0.13	1.04	1.84	0.34	212	302
RP Bench	0.20	2.40	0.07	-0.13	1.04	1.72	0.10	475	416

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Part 2: Feature selection

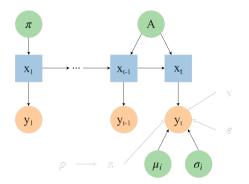
- Traditionally, features to train HMMs are selected in advance (either expert knowledge or data availability). However, these features don't necessarily contribute to regime identification, which is our goal.
- In ML it is customary to use feature selection to improve model performance.
- However unsupervised feature selection research for HMMs is limited.

We implemented an embedded feature selection algorithm based on: *Feature Selection for Hidden Markov Models and Hidden Semi-Markov Models* Adams et al. (2016)

Feature Saliency HMM

The idea is to divide features into two groups,

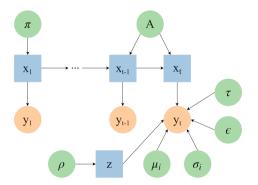
- state dependent (good features), modelled by Gaussians that are state dependent with parameters μ_i and σ_i
- state independent (irrelevant features), modelled by gaussians with parameters τ and ϵ .
- ρ (feature saliency) is the probability that the feature is relevant.



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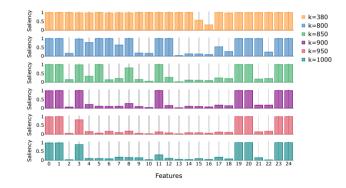
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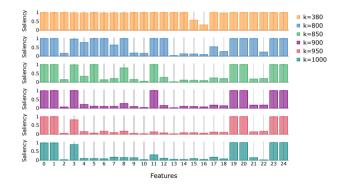
Relevant Features

• k is a hyper-parameter that can be used as cost of the feature. The higher its value, the less features will be selected.



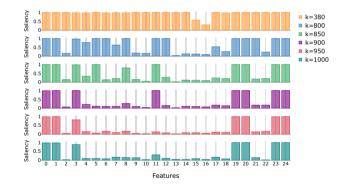
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Relevant Features

- k is a hyper-parameter that can be used as cost of the feature. The higher its value, the less features will be selected.
- We can see that the selected parameters are stable, and after a value of *k* above 850, the number of features doesn't change much.
- Selected factors are: Book Value Yield, 1 Yr Fwd Earnings Yield, Sales Yield, 6 Month Price Momentum, 12 Month Price Momentum, EPSCV, Beta.



DAA system with FSHMM

To test the DAA system incorporating the FS algorithm, we built two cases:

- Trained one HMM with all 25 features.
- Trained one HMM with the relevant subset of features.

FSHMM tends to be more sensible to the distress state - it spends 24% of the time in this state versus 20% of the model trained with full set of features.

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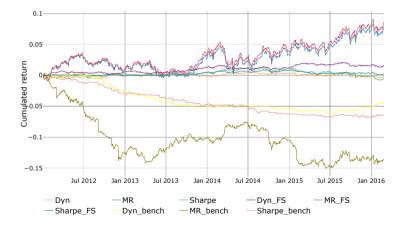
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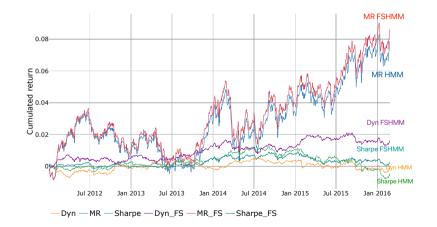
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Same DAA system as before, with daily evaluation of the state and monthly retraining.

- For the allocation, we used MSCI factor indices, so we had to estimate the mean and covariance for each regime.
- We built Dynamic, Max return and Sharpe portfolios only.





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	Ann ret	Ann vol	IR	Skw	kurt	D. risk	Sortino	DD	DD days
Sharpe FSHMM	0.061	0.50	0.12	-0.71	2.85	0.37	0.16	-94	387
Sharpe HMM	-0.11	0.65	-0.16	-0.70	3.84	0.49	-0.22	-164	522
Sharpe Bench	-1.62	0.92	-1.76	-2.75	15.0	0.82	-1.98	19825	1452
Dyn FSHMM	0.39	0.65	0.61	-0.41	0.84	0.47	0.84	-52	141
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MSCI Quality	0.50	2.76	0.18	0.20	2.02	1.90	0.26	-208	837
MSCI Enhanced Value	0.03	3.97	0.01	0.029	0.86	2.83	0.01	-105	599
MSCI High Dividend Yield	-2.16	3.22	-0.67	0.38	0.85	2.24	-0.96	-2374	1317
MSCI Momentum	2.48	4.35	0.57	-0.35	1.42	3.11	0.80	-144	475
MSCI Minimum Volatility	-0.89	3.58	-0.25	0.10	0.69	2.52	-0.35	-38371	906
MSCI Equal Weighted	-0.27	2.94	-0.09	-0.045	0.74	2.09	-0.13	-135	675

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MR HMM	1.85	3.19	0.58	-0.39	1.84	2.29	0.80	-92	62
MR Bench	-3.46	3.78	-0.91	-2.71	20.5	3.17	-1.09	-4032	1250
MSCI Quality	0.50	2.76	0.18	0.20	2.02	1.90	0.26	-208	837
MSCI Enhanced Value	0.03	3.97	0.01	0.029	0.86	2.83	0.01	-105	599
MSCI High Dividend Yield	-2.16	3.22	-0.67	0.38	0.85	2.24	-0.96	-2374	1317
MSCI Momentum	2.48	4.35	0.57	-0.35	1.42	3.11	0.80	-144	475
MSCI Minimum Volatility	-0.89	3.58	-0.25	0.10	0.69	2.52	-0.35	-38371	906
MSCI Equal Weighted	-0.27	2.94	-0.09	-0.045	0.74	2.09	-0.13	-135	675

	Ann ret	Ann vol	IR	Skw	kurt	D. risk	Sortino	DD	DD days
Sharpe FSHMM	0.061	0.50	0.12	-0.71	2.85	0.37	0.16	-94	387
Sharpe HMM	-0.11	0.65	-0.16	-0.70	3.84	0.49	-0.22	-164	522
Sharpe Bench	-1.62	0.92	-1.76	-2.75	15.0	0.82	-1.98	19825	1452
Dyn FSHMM	0.39	0.65	0.61	-0.41	0.84	0.47	0.84	-52	141
Dyn HMM	-0.02	0.60	-0.03	-1.12	9.03	0.45	-0.04	-175	566
Dyn Bench	-1.10	1.03	-1.07	-2.76	16.2	0.88	-1.24	-1508	1123
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- We incorporated embedded feature selection algorithm to our systematic trading framework. This improves model's accuracy and allows for a more objective approach.
- We tested both models using MSCI USA enhanced factor indices. Portfolios constructed using feature saliency HMM show a higher performance than the same portfolios constructed using full-feature HMM.

Future Work

Include macroeconomic series in the training, where the embedded feature selection could potentially solve the problem of selecting relevant economic series.

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- **2** This would be particularly interesting for other asset classes such as fixed income.
- A drawback of using HMMs is selecting the number of latent states beforehand, we could address this using an infinite HMM.

Thank you!



Paper pre-print: https://arxiv.org/abs/1902.10849 or scan QR code!

https://github.com/elifons/FeatureSaliencyHMM